

Soft pions at high energy and the flavor asymmetry of the light sea quarks in the nucleon

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The modified Gottfried sum rule makes clear importance of the high energy region not only in the theoretical meaning but also in the numerical analysis. In this talk, it is shown that the soft pion theorem in the inclusive reaction at high energy can explain the magnitude about $0.02 \sim 0.04$ in the NMC deficit. The main contribution comes from the small x region. We also estimate the soft pion contribution to the Ellis-Jaffe sum rule and show it to be negligible. However we find that the contribution to g_1^{ep} becomes positive below $x = 0.002$.

1 Introduction

The modified Gottfried sum rule [1] has explained the NMC deficit in the Gottfried sum [2] almost model independently. It has shown that the deficit is the reflection of the hadronic vacuum originating from the spontaneous chiral symmetry breaking. In this sense the physics underlining this algebraic approach has a common feature with that of the mesonic models reviewed in Ref.[3]. However, in the algebraic approach, importance of the high energy region not only in the theoretical meaning but also in the numerical analysis has been made clear. Further the numerical prediction based on this sum rule exactly agrees with the recent experimental value from E866/NuSea collaboration [4]. This experiment also gives us the light antiquark difference ($\bar{d}(x) - \bar{u}(x)$) and the ratio $\bar{d}(x)/\bar{u}(x)$ in the range $0.02 \leq x \leq 0.345$. An unexpected behavior is that the asymmetry seems to disappear at large x . On the other hand, a typical calculation in the mesonic models based on the πNN and the $\pi N\Delta$ processes account for about a half of the NMC deficit [3]. According to the E866 experiment, an explanation of the remaining half of the NMC deficit should be given by contributions in the small x region. Unfortunately, the approach from the mesonic models can not account for the magnitude from these regions definitely. In fact, the $\pi N\Delta$ process partly cancels the positive contribution to the ($\bar{d}(x) - \bar{u}(x)$) from the πNN process. The contributions from the higher resonances or from the multiparticle states are obscure. Hence the best we can say is that the mesonic models explain the flavor asymmetry of the light sea quarks qualitatively. These facts suggest that there may exist a dynamical mechanism so far overlooked to produce the flavor asymmetry at medium and high energy, and that it may compensate the above flaw of the mesonic models. In this talk, it is shown that the soft pion theorem in the inclusive reaction at high energy [5] can explain the magnitude about $0.02 \sim 0.04$ in the NMC deficit. The theoretical prediction depends heavily on the spin dependent structure function ($g_1^{ep} - g_1^{en}$), and the main contribution to the NMC deficit in this theorem comes from the small x region. Thus, this type of the contribution may become the background of the mesonic model.

2 Soft pion at high energy

Here we briefly explain the soft pion theorem in the inclusive reactions. Usually, the soft pion theorem has been considered to be applicable only in the low energy

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regions. However in Ref.[5], it has been found that this theorem can be used in the inclusive reactions at high energy if the Feynman's scaling hypothesis holds. In the inclusive reaction " $\pi + p \rightarrow \pi_s(k) + \text{anything}$ " with the π_s being the soft pion, it states that the differential cross-section in the center of the mass (CM) frame defined as

$$f(k^3, \vec{k}^\perp, p^0) = k^0 \frac{d\sigma}{d^3k}, \quad (1)$$

where p^0 is the CM frame energy, scales as

$$f \sim f^F\left(\frac{k^3}{p^0}, \vec{k}^\perp\right) + \frac{g(k^3, \vec{k}^\perp)}{p^0}. \quad (2)$$

If $g(k^3, \vec{k}^\perp)$ is not singular at $k^3 = 0$, we obtain

$$\lim_{p^0 \rightarrow \infty} f^F\left(\frac{k^3}{p^0}, \vec{k}^\perp = 0\right) = f^F(0, 0) = \lim_{p^0 \rightarrow \infty} f(0, 0, p^0). \quad (3)$$

This means that the π mesons with the momenta $k^3 < O(p^0)$ and $\vec{k}^\perp = 0$ in the CM frame can be interpreted as the soft pion. This fact holds even when the scaling violation effect exists, since we can replace the exact scaling by the approximate one in this discussion. The important point of this soft pion theorem is that the soft-pion limit can not be interchanged with the manipulation to obtain the discontinuity of the reaction " $a + b + \pi_s \rightarrow a + b + \pi_s$ ". We must first take the soft pion limit in the reaction " $a + b \rightarrow \pi_s + \text{anything}$ ". This is because the soft pion attached to the nucleon (anti-nucleon) in the final state is missed in the discontinuity of the soft pion limit of the reaction " $a + b + \pi_s \rightarrow a + b + \pi_s$ " [5].

3 Kinematics

Now we consider the soft pion theorem in the semi-inclusive currents-nucleon reaction with use of the light cone variable [6]. We take the electromagnetic current for an illustration. We first take $k^+ = 0$ and $\vec{k}^\perp = 0$, and then take $k^- = 0$. In this limit, $k^2 = 0$ but the momentum of the virtual γ and the nucleon are unrestricted. The amplitude in this limit is classified into three terms as is shown in Fig.1. The graph (a) is the contribution from the pion emission from the initial nucleon, the graph (b) is the one from the final nucleon or the anti-nucleon, and the graph (c) is the term coming from the null-plane commutation relation at $x^+ = 0$. Then the hadronic tensor can be obtained by squaring the amplitude and the typical graphs contributing to this tensor are shown in Fig.2. We discard the graphs (e)~(h) in Fig.2. These graphs are characterized by the one soft pion from the nucleon (anti-nucleon) in the final state. Because this emission vertex is proportional to the helicity of the nucleon (anti-nucleon), the positive nucleon (anti-nucleon) and the negative one in the final state cancels each other at high energy, while at low energy these graphs are suppressed in the deep inelastic region by the form factor effects. Among the remaining graphs, the graphs (a) and (d) are directly related to the known processes. However the graphs (b),(c), and (i) can not in general be related to the known processes without further assumptions. Now in the deep inelastic region, these graphs are light-cone dominated, hence we can use the perturbative QCD such as the cut vertex formalism[7]. To obtain the relation to the structure functions in the total inclusive reactions, however, we must use the inverse transform of the moment sum rules. The net result of such an analysis can be obtained by using the light-cone current algebra [8] at some initial $Q^2 = Q_0^2$ where the evolution is started and then to take the Q^2 dependence into account through the structure functions related by this way.

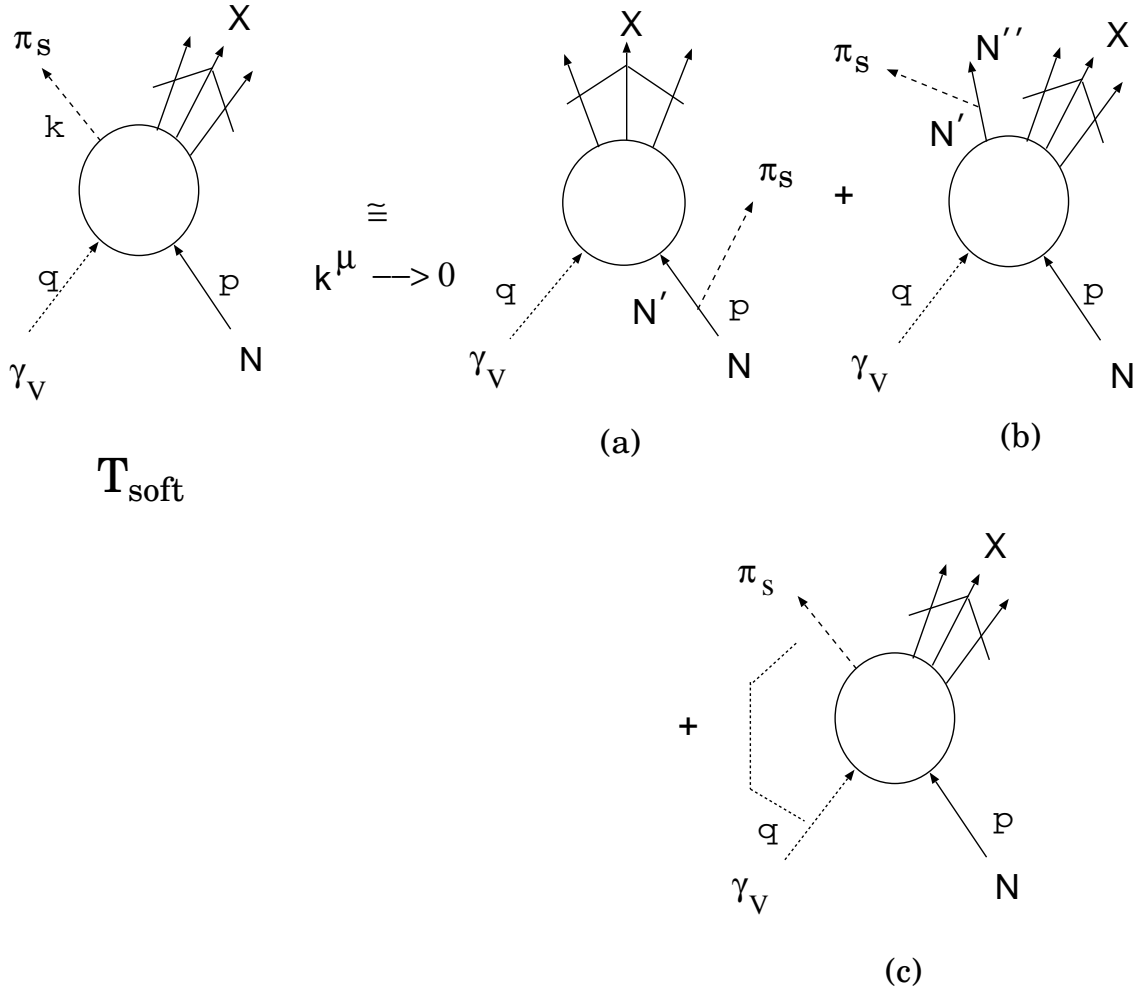


Figure 1: Soft pion theorem in the inclusive reaction. the graph (a) corresponds to the pion emission from the initial nucleon, the graph (b) corresponds to the final nucleon or the anti-nucleon, and the graph (c) corresponds to the term coming from the null-plane commutator.

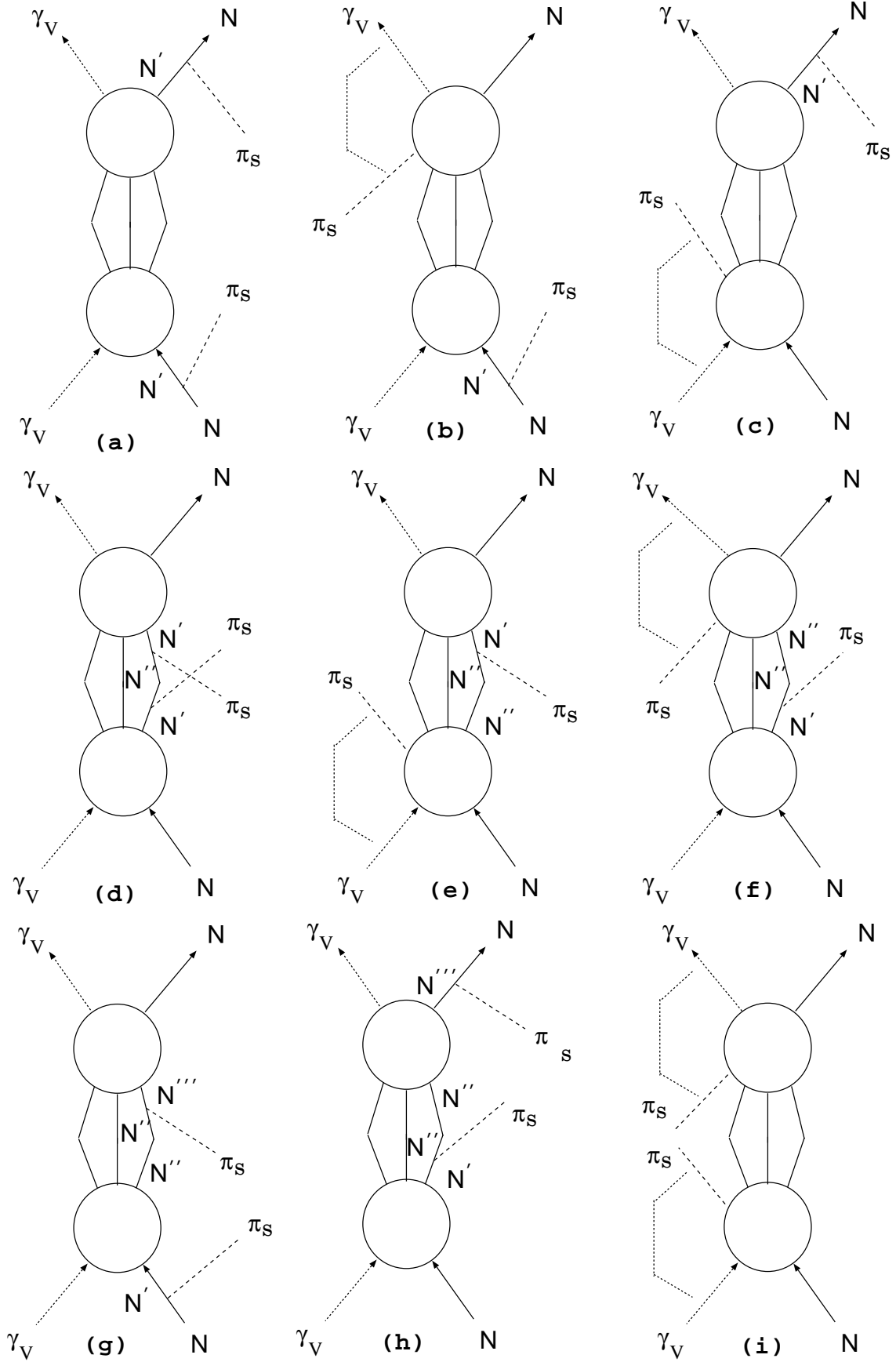


Figure 2: The graphs contributing to the hadronic tensor

4 The charge asymmetry

The method in Ref.[5] had not been checked experimentally, hence it was done in the soft π^- case [9]. From the experimental data of the Harvard-Cornell group[10] the data satisfying the following conditions

- (1) The transverse momentum satisfies $|\vec{k}^\perp|^2 \leq m_\pi^2$.
- (2) The change of F^- can be regarded to be small in the small x_F region.

are selected. The effective cut of x_F is about 0.2. Then the theoretical value is about 10% \sim 20% of the experimental value. However, in the central region, there are many pions from the decay of the resonances, and about 20% \sim 30% can be expected to be the pion from the directly produced pion. Hence the theoretical value is the same order with the experimental value. Now to reduce the ambiguity due to the pion from the resonance decay product, the charge asymmetry was calculated[11]. By assuming symmetric sea polarization for simplicity, we obtain

$$F_2^+ - F_2^- = \frac{1}{4f_\pi^2} [F_2^{\bar{\nu}p} - F_2^{\nu p} + 2g_A^2(0)F_2^{en} - 8xg_A(0)(g_1^{ep} - g_1^{en}) + 2g_A^2(0)(\langle n \rangle_p + \langle n \rangle_{\bar{n}} - \langle n \rangle_{\bar{p}} - \langle n \rangle_n)F_2^{ep}], \quad (4)$$

$$F^+ - F^- = \frac{1}{64\pi^3 f_\pi^2} \left[\frac{F_2^{\bar{\nu}p} - F_2^{\nu p}}{F_2^{ep}} + 2g_A^2(0) \frac{F_2^{en}}{F_2^{ep}} - 8xg_A(0) \frac{(g_1^{ep} - g_1^{en})}{F_2^{ep}} + 2g_A^2(0)(\langle n \rangle_p + \langle n \rangle_{\bar{n}} - \langle n \rangle_{\bar{p}} - \langle n \rangle_n) \right]. \quad (5)$$

where F^α is defined as

$$F^\alpha = \frac{1}{\sigma_T} q^0 \frac{d\sigma^\alpha}{d^3q}. \quad (6)$$

By neglecting the nucleon multiplicity term which can be expected to be a small positive contribution, the theoretical value is roughly equal to 0.15 \sim 0.18. While the experimental value with the transverse momentum satisfying $|\vec{k}^\perp|^2 \leq m_\pi^2$ in Ref.[10] is almost constant in the region $0 < x_F < 0.1$ with its value 0.28 ± 0.05 , and it gradually decreased above $x_F = 0.1$. Hence the theoretical value is very near to the experimental value.

5 The soft pion contribution to the Gottfried sum

The charge asymmetry in the central region may contribute to the Gottfried sum and hence its magnitude has been estimated [12]. Adding the contributions from the soft π_s^+ , π_s^- , and π_s^0 , and subtracting the contributions to F_2^{en} from those to F_2^{ep} , we obtain

$$(F_2^{ep} - F_2^{en})|_{soft} = \frac{I_\pi}{4f_\pi^2} [g_A^2(0)(F_2^{ep} - F_2^{en})(3\langle n \rangle - 1) - 16xg_A(0)(g_1^{ep} - g_1^{en})], \quad (7)$$

where I_π is the phase space factor for the soft pion defined as

$$I_\pi = \int \frac{d^2\vec{k}^\perp dk^+}{(2\pi)^3 2k^+}, \quad (8)$$

and $\langle n \rangle$ is the sum of the nucleon and anti-nucleon multiplicity defined as $\langle n \rangle = \langle n \rangle_p + \langle n \rangle_n + \langle n \rangle_{\bar{p}} + \langle n \rangle_{\bar{n}}$. Note that the spin dependent

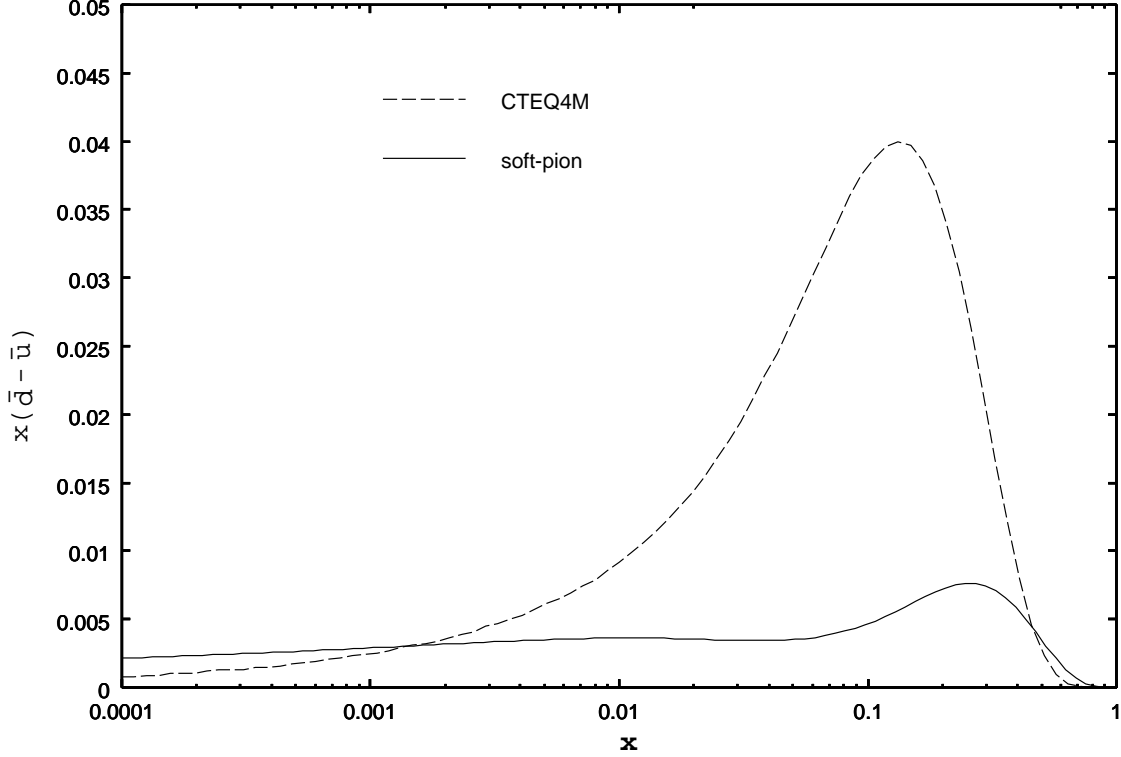


Figure 3: Soft pion contribution to the flavor asymmetry of the light sea quarks

term is obtained by assuming the symmetric sea polarization for simplicity. We can express $(F_2^{ep} - F_2^{en})|_{soft}$ as the asymmetry of the antiquark distribution as

$$(F_2^{ep} - F_2^{en})|_{soft} = -\frac{2}{3}x(\bar{d} - \bar{u})|_{soft}. \quad (9)$$

To estimate the magnitude of this asymmetry, we approximate $F_2^{ep}, F_2^{en}, g_1^{ep}, g_1^{en}$ on the right-hand side of Eq.(5.1) by the valence quarks distribution functions at $Q_0^2 = 4 \text{ GeV}^2$ [13]. As a multiplicity of the nucleon and antinucleon, we set

$$\langle n \rangle = a \log_e s + 1, \quad (10)$$

where $s = (p + q)^2$. The parameter a is fixed as 0.2 in consideration for the proton and the anti-proton multiplicity in the e^+e^- annihilation such that $\frac{1}{2}a \log_e s$ with \sqrt{s} replaced by CM energy of that reaction agrees with it [14]. Following the experimental check of the charge asymmetry in the previous section, the transverse momentum is restricted by the condition (1) and the Feynman scaling variable is cut at $x_F = 0.1$. By calculating the phase space factor under these conditions and allowing a small change of the parameters to determine the phase space, it is shown that we can expect the magnitude of the contribution to the Gottfried sum from the soft pion is about $-0.04 \sim -0.02$. The main contribution comes from the small x region as is shown in Fig.3, where we give a plot of the same asymmetry given by CTEQ4M[15] for comparison.

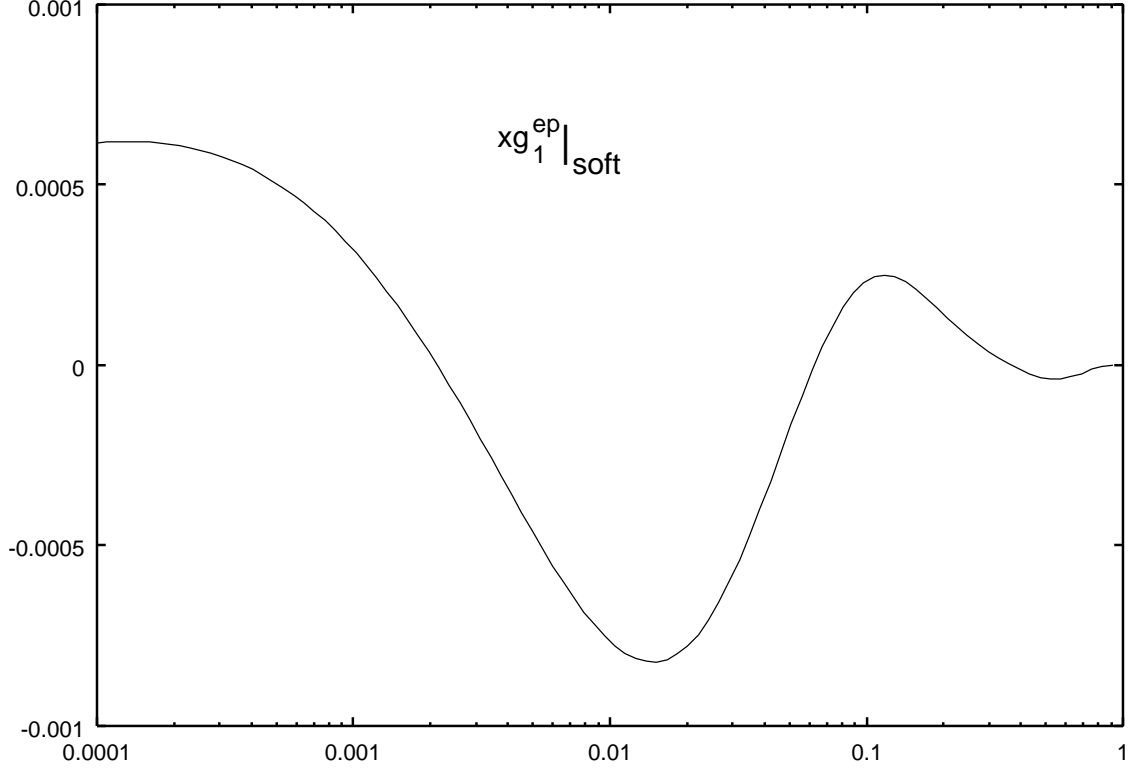


Figure 4: Soft pion contribution to the xg_1^{ep}

6 The soft pion contribution to the Ellis-Jaffe sum rule

The soft pion contribution to the g_1^{ep} is estimated under the same condition in the previous section, and we obtain

$$xg_1^{ep}|_{soft} = \frac{I_\pi}{4f_\pi^2} [-(xg_1^{\nu p} + xg_1^{\bar{\nu} p}) + g_A^2(0)(xg_1^{ep} + 2xg_1^{en}) + 3g_A^2(0) \langle n \rangle xg_1^{ep} - 3g_A(0)(F_2^{ep} - F_2^{en}) - \frac{g_A(0)}{6}(F_2^{\nu p} - F_2^{\bar{\nu} p})]. \quad (11)$$

Using the same quark distribution functions as in the previous section, the right-hand side of Eq.(6.1) is estimated and the result is shown in Fig.4. The contribution above $x = 0.001$ is small, and we see that the contribution from the small x region may become large. However the effect of the soft pion above $x = 0.0001$ to the Ellis-Jaffe sum rule is negligible.

7 Summary

We have calculated the soft pion contribution to the Gottfried sum and find that its magnitude is about $-0.04 \sim -0.02$. This magnitude is about the same order required by the meson cloud model. The same soft pion contribution to the Ellis-Jaffe sum rule in the region above $x = 0.0001$ is calculated, and find that it is very small, but we find that the soft pion contribution to xg_1^p below $x = 0.002$ becomes positive.

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